Algerian Mathematical Olympiad - Second Edition 2025

Category: Benjamin <div style="text-align: right">**July 3rd, 2025**</div>

Problem 1. Find all natural numbers n such that : $\dfrac{n^6 + n + 61}{n^2 + n + 1}$ is an integer.

Problem 2. Find all positive real numbers $a_1, a_2, ..., a_{45}$ such that:

$$\sum_{k=1}^{45} k^2 a_k + \frac{1}{2025 a_k} = 46$$

Problem 3. Let ABC be a right triangle in A. Let ω be its circumcircle and D the midpoint of BC. Let E be the intersection of AD with ω and let F be the intersection of the perpendicular bisector of AC with (BDE). The line CF intersects (BDE) in P and the let Q be the intersection of DP with AB. Let S and T be the symmetric points of P and F with respect to D respectively.
Prove that the intersection point of the lines AT and QF is the circumcenter of ΔQAS.