People's Democratic Republic of Algeria
Ministry of National Education
Directorate of Specialized Education and Private Education
National Committee of Olympiads of Educational Disciplines

Algerian Mathematical Olympiad - Second Edition 2025

Category: Senior **July 4th, 2025**

Problem 4. Prove that for all strictly positive reals $x, y, z \in \mathbb{R}$ such that $xy+yz+zx = 3$ we have:

$$\frac{1}{xyz} \geq \frac{2}{3}(\sqrt{x} + \sqrt{y} + \sqrt{z}) - 1$$

Problem 5. Let N be a positive integer and let $a_1, a_2, \ldots, a_N$ be positive integers.
Define for all $n > N$, a_n to be the least occurring integer in $a_1, a_2, \ldots, a_{n-1}$(if the smallest number of occurrences is shared between two distinct integers a_k, let a_n be the smaller of the two).
Prove that the sequence $(a_n)_{n\in\mathbb{N}^*}$ is eventually periodic.

Example : For $N = 5$, for

$$a_1 = 1, a_2 = 6, a_3 = 8, a_4 = 3, a_5 = 1$$

we get: $a_6 = 3, a_7 = 6, a_8 = 8, \ldots$

Example : the following sequence is periodic starting from the fourth term and its period is 2:

$$5, 4, 3, 1, 2, 1, 2, 1, 2, \ldots$$

Problem 6. Let ABC be a scalene triangle with circumcircle ω. The perpendicular bisector of the segment $[AB]$ intersects (BC) and (AC) in D and E, respectively, such that E lies outside $[AC]$. The perpendicular to (BC) from D intersects (BCE) at a point X outside ΔABC. Line (DX) intersects (AC) at Y and ω in Z and T such that Z lies on the arc AC that does not contain B. The circumcircle of triangle ΔZET intersects the line (BC) and the circumcircle of ΔYDE in P and Q, respectively.
Prove that the tangent to (YZQ) from Z, the tangent to (YTQ) from T, and the line (PX) meet at one point.

Language: English

Time: 4 hours and 30 minutes
Each problem is worth 7 points.
The problems are ordered by difficulty.